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Dynamical properties of speckled speckles

Steen G. Hanson[1], Theis Faber Quist Iversen[1] and René Skov Hansen[2]
1. DTU Fotonik, Department of Photonics Engineering
DK-4000 Roskilde, Denmark
2. University of Southern Denmark, 5230 Odense M, Denmark

ABSTRACT

We consider the dynamical properties of speckles observed through a second static diffuser arising from a linear or angularly displaced first diffuser. Analytical expressions are obtained for general situations where both the space between the displaced and the static diffuser and the space between the static diffuser and the plane of observation consist of an optical system that can be characterized by a complex-valued *ABCD*-matrix (e.g. simple and complex imaging systems, free space propagation in both the near- and far-field, and Fourier transform systems). The use of the complex *ABCD*-method means that diffraction due to inherent apertures is included. One of the diffusers is assumed to give rise to fully developed speckle, i.e. the scattered phase is assumed to be delta-correlated, whereas the second and dynamic diffuser has a finite lateral scale. The illumination of the displaced diffuser is assumed to be Gaussian but the derived expressions are not restricted to a plane incident beam. The results are applicable for speckle-based systems for determining mechanical displacements, especially for long-range systems, and for analyzing systems for measuring biological activity beyond a diffuse layer, e.g. blood flow measurements through human skin.

Key words: speckle dynamics, velocity measurement, speckled speckles, bio speckles, tissue optics

1. INTRODUCTION

The use the granularity structure named speckles that arise when coherent light is scattered from a rough surface and the intensity is probed has been extensively studied [1]. When the scattering surface is undergoing a displacement, the observed speckle pattern, itself, will show dynamic properties. This fact has been used intensively for various applications[2]. Of special interest for the present evaluation is the determination of velocity of structures buried in a diffuse medium, here modelled as an illuminated dynamic structure giving rise to speckle but buried behind a diffuse screen. The issue of speckled speckles has previously been addressed. Françon *et al*.[3] have experimentally shown how minute displacements can be monitored, while Iwai and Asakura[4] experimentally have investigated triply-scattered speckles for displacement measurement. Here, the incident light first passed a diffuser, after which the speckled light hit the dynamic structure, after which the scattered light passed the first diffuser and was detected. Also in the case of laser eye safety, the issue of speckled speckles is of importance[5]. In early investigations of the phenomenon of speckled speckles, experiments were conducted, in which the object was displaced with a constant velocity, and the spectral temporal content of the doubly scattered light was examined, especially with respect to its correlation decay time[4,6-11]. In this case, the distinction between speckle decorrelation ("boiling") and speckle displacement were mixed with the speckle size, giving little information on each individual component.
A specific reason for many of these investigations has been the investigation of optically based systems for probing sub-cutaneous blood flow. Ruth[12] has very early shown how it is possible based on the dynamical speckled speckle pattern to provide information on the embedded blood flow. Of special interest in this evaluation was the exclusion of the speckle

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dynamics from the inadvertent movement of the skin, in order to enhance and separate the information from the blood flow. Later, investigations have been conducted showing measurements of blood flow[13,14] and for measuring skin thickness, as well[15].

To support the experimental findings, we will in the following sketch the derivation of speckle size, decorrelation length, and gearing of speckle displacement with relation to object displacement in case of speckled speckles. Furthermore, we will derive analytical expressions in case the two diffusers are separated by free space, and the speckles arising from the second, and static, diffuser are observed after propagation in free space. The expressions will cover both the near field and the far field. No polarization effects will be included.

We have used the generalized ray-matrix formulation[16] to derive an expression for the time- and space-lagged crosscovariance of the intensity distribution that arises when diffusely scattered coherent light passes an optical system followed by another diffuser, after which a second optical system transfers the light to the detector plane. Following the theoretical evaluation, the results for free space propagation will be supported by experiments. Conclusions will be presented, highlighting the applicability of speckle speckles and showing issues related to probing sub-cutaneous blood flow.

2. THEORETICAL DESCRIPTION

The set-up shown in figure 1 will be analyzed. Here a diffusely scattering object is illuminated with temporally and spatially coherent light.

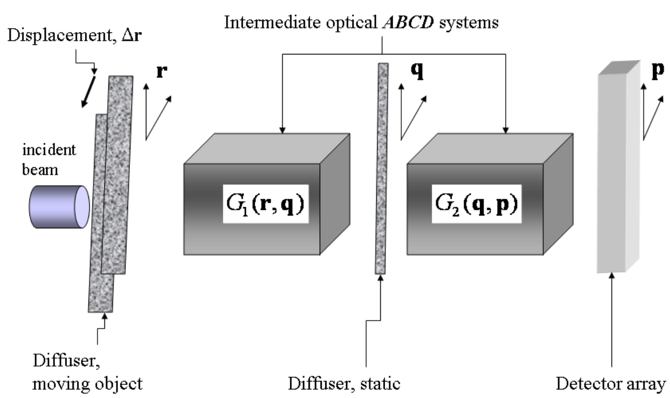


Figure 1. The setup used for analyzing the dynamics for speckled speckles

An incident Gaussian beam illuminates the rough object surface essentially which produces a fully developed speckle field, although - as will be explained later – it is mimicked as a diffuser giving rise to partially developed speckle. In the analysis, the incident illumination beam is assumed collimated with a beam width ω_0 (1/e^2-radius). An arbitrary incident beam curvature can be incorporated as a first optical element in the Green's function $G_1(\mathbf{r},\mathbf{q})$ connecting the first dynamic diffuser and the second static diffuser. The scattered light from the diffuse object is transformed by the complex *ABCD* system with the Green's function $G_1(\mathbf{r},\mathbf{q})$ connecting the input plane r with the intermediate plane q, where a second diffuser is placed. A second complex *ABCD* system with a Green's function $G_2(\mathbf{q},\mathbf{p})$ transforms the field distribution from plane **q** to plane **p**, where a 2D detector array records the intensity distribution. A recording of the

speckle pattern is made before and after an in-plane displacement $\Delta\mathbf{r}_0$ of the diffuse object has taken place. The time/space-lagged cross correlation between the two intensity recordings is performed in order to deduce the speckle size, the speckle displacement, and the speckle decorrelation that has taken place during the translation of the object.
The connection between the fields in the two planes is given by

$$U_{2,in}(\mathbf{q})=\int d^2\mathbf{r}\; G_1(\mathbf{r},\mathbf{q})U_{1out}(\mathbf{r}),\; U_{2,out}(\mathbf{q})=\Psi(\mathbf{q})\; U_{2,in}(\mathbf{q}),\text{ and } U_3(\mathbf{p})=\int d^2\mathbf{q}\; G_2(\mathbf{q},\mathbf{p})U_{2,out}(\mathbf{q})\,, \tag{1}$$

where the indices in and out denote the field before and after passing the respective diffuser. The Green's function for each of the two intermediate optical systems are given by

$$G_{1,2}(\mathbf{r},\mathbf{p})=-\frac{ik}{2\pi B_{1,2}}\exp\left[-\frac{ik}{2B_{1,2}}(A_{1,2}\mathbf{r}^2-2\mathbf{r}\cdot\mathbf{p}+D_{1,2}\mathbf{p}^2)\right]. \tag{2}$$

In general, the matrix elements are complex valued if the train of optical components includes limiting apertures, in this formalism assumed Gaussian apodized[16]. In writing the Green's functions as shown above, it has been assumed that the refractive index in the input and output plane is identical and that the optical systems are rotationally symmetric. In order to arrive at results that are easily intelligible, we will in the following derivation assume all matrix elements to be real-valued. The expansion to include complex-valued matrix elements is straight-forward following the same path as shown here.

The field incident on the q-plane is given by

$$U_{2in}(\mathbf{q})=\int d^2\mathbf{r}\; G_1(\mathbf{r},\mathbf{q})\exp\left[-i\varphi_1(\mathbf{r})\right]U_0(\mathbf{r}). \tag{3}$$

where $\varphi_1(\mathbf{r})$ is the phase perturbation from the first dynamic diffuser and $U_0(\mathbf{r})$ is the incident field. In like manner, we can find the field incident on the detector array:

$$U_3(\mathbf{p})=\int_{-\infty}^{\infty}\int d^2\mathbf{r}\; d^2\mathbf{q}\; G_1(\mathbf{r},\mathbf{q})G_2(\mathbf{q},\mathbf{p})\exp\left[-i\varphi_1(\mathbf{r})\right]\exp\left[-i\psi_2(\mathbf{q})\right]U_0(\mathbf{r}), \tag{4}$$

where $\psi_2(\mathbf{q})$ is the phase perturbation from the second static diffuser. The spatial structure of the incident field is given by

$$U_0(\mathbf{r})\propto\exp\left[-\frac{\mathbf{r}^2}{\omega_0^2}\right], \tag{5}$$

and the phase correlations for the two diffusers are given by

$$\begin{gathered}\left\langle\exp\left[-i\varphi_1(\mathbf{r}_1)\right]\exp\left[i\varphi_1(\mathbf{r}_2-\Delta\mathbf{r}_0)\right]\right\rangle\propto\exp\left[-\frac{\left(\mathbf{r}_1-\mathbf{r}_2-\Delta\mathbf{r}_0\right)^2}{\rho_1^2}\right]\;.\\ \text{and }\left\langle\exp\left[-i\psi_2(\mathbf{q}_1)\right]\exp\left[i\psi_2(\mathbf{q}_2)\right]\right\rangle\propto\delta\left(\mathbf{q}_1-\mathbf{q}_2\right)\end{gathered} \tag{6}$$

We have here included the induced displacement $\Delta\mathbf{r}_0$ introduced to the first diffuser between the two exposures by the CCD-array. The ensemble average is defined by angular brackets. We have in the above and in the remainder neglected the scaling of the absolute amplitude values, as we are interested only in the dynamical properties. The *ABCD*-matrix method only holds for paraxial systems; and in the treatment to follow, we will violate this restriction by reducing the effective distance, i.e. the *B*-element, to be comparable with the incident spot size ω_0 . In fact, the irradiance distribution after a diffuser giving rise to fully developed speckles will make the irradiance in an adjacent plane cover the entire half plane, which is not valid due to the violation of the Fresnel approximation and the angular intensity distribution after the diffuser. In order to cope with this, a synthetic scale being close to the wavelength ρ_1 for the first diffuser has been included.

Of interest for the following will be the intensity covariance, $\Gamma[\mathbf{p};\mathbf{p}+\Delta\mathbf{p}]$, in the detector plane before- and after a displacement of the object has taken place, i.e.

$$\begin{aligned}\Gamma[\mathbf{p};\mathbf{p}+\Delta\mathbf{p}] \equiv \langle I_3(\mathbf{p}) I_3'(\mathbf{p}+\Delta\mathbf{p})\rangle - \langle I_3(\mathbf{p})\rangle\langle I_3'(\mathbf{p}+\Delta\mathbf{p})\rangle = \\ \left|\langle U_3^*(\mathbf{p}) U_3'(\mathbf{p}+\Delta\mathbf{p})\rangle\right|^2\end{aligned} \quad . \tag{7}$$

Here primed indices indicate a displaced first diffuser, and we have assumed circular symmetric Gaussian statistics for the output field (Siegert relation).

In case of no displacement of the first diffuser, we get the normalized auto covariance of the fields

$$\begin{aligned}\gamma_{static}(\Delta\mathbf{p},\mathbf{P}) = \frac{\langle U_3(\mathbf{P}) U_3^*(\mathbf{P}+\Delta\mathbf{p})\rangle}{\sqrt{I(\mathbf{P}) I((\mathbf{P}+\Delta\mathbf{p})}} = \exp\left[-\frac{\Delta\mathbf{p}^2}{8}\left(\frac{8B_1^2}{B_2^2\rho_1^2} + \frac{A_1^2 k^2 \omega_0^2}{B_2^2}\right)\right]\exp\left[-\frac{ikD_2}{B_2}\mathbf{P}\cdot\Delta\mathbf{p}\right]. \\ \text{where } \mathbf{p}_1 \to \mathbf{P} + \frac{\Delta\mathbf{p}}{2} \text{ and } \mathbf{p}_2 \to \mathbf{P} - \frac{\Delta\mathbf{p}}{2}\end{aligned} \tag{8}$$

The absolute value of the auto covariance only depends on the relative distance $\Delta\mathbf{p}$ in the observation plane (CCD array), whereas the phase of the auto covariance is space dependent through $\mathbf{P}$. The speckle size for static speckled speckles is defined by the $\Delta\mathbf{p}$-value for which the covariance has decreased to $1/e^2$. In arriving at the above expression, we have assumed $\rho_1 << \omega_0$.

In case of translating the first diffuser a distance $\Delta\mathbf{r}_0$, the time lagged spatial covariance can be derived by inserting the expressions in Eqs. (5) and (6) into Eq. (4). We get after some calculations:

$$\left|\gamma_{dynamic}(\Delta\mathbf{p},\mathbf{P})\right|^2 = \exp\left[-\Delta\mathbf{p}^2\frac{A_1^2 k^2\omega_0^2}{4B_2^2}\right]\exp\left[-\frac{\Delta\mathbf{r}_0^2}{\omega_0^2}\left(1-\frac{A_1^2 k^2 \rho_1^2}{8B_1^2/\omega_0^2 + A_1^2 k^2 \rho_1^2}\right)\right]\exp\left[-\frac{2\left(\Delta\mathbf{r}_0 - \Delta\mathbf{p}\ B_1/B_2\right)^2}{\rho_1^2}\right] \tag{9}$$

The above is the main result of this investigation, as it reveals the speckle size, the decorrelation effects and the gearing between the displacement of the first diffuser and the observed speckle displacement. And, needles to say, the result is valid for any optical system within the *ABCD*-formalism being it imaging, Fourier transform, or free space propagation being it for the first $G_1(\mathbf{r},\mathbf{q})$ or the second optical system $G_2(\mathbf{q},\mathbf{p})$. It should be emphasized that the above expression indicates that the peak of the covariance appears for $\Delta\mathbf{p} = \Delta\mathbf{r}_0\, B_2/B_1$, which is not correct, as the two decorrelation terms tend to "drag" the peak position. The correct position for the peak of the covariance is

$$\Delta\mathbf{p}_{peak} = \frac{8B_1B_2}{8B_1^2 + A_1^2 k^2 \rho_1^2 \omega_0^2}\Delta\mathbf{r}_0\ , \tag{10}$$

which defines the gearing of the speckles as a function of the object displacement. The decorrelation, i.e. the decrease of the value of the covariance at the peak position becomes:

$$\Delta\mathbf{r}_{0,dec} = \omega_0\sqrt{\frac{8B_1^2 + A_1^2 k^2 \rho_1^2 \omega_0^2}{8B_1^2 + 2A_1^2 k^2 \omega_0^4}}. \tag{11}$$

Likewise, the speckle size in the observation plane turns out to be

$$\rho_{speckle} = \frac{2 B_2 \rho_1}{\sqrt{8 B_1^2 + A_1^2 k^2 \rho_1^2 \omega_0^2}} . \tag{12}$$

It is noted that none of the above three important parameters depend on the A_2- and D_2-elements due to the assumption of the second diffuser giving rise to fully developed speckle. Only the effective length of the second optical system, i.e. the value of B_2 affects these parameters.

One of the issues for this investigation was to investigate to what extent the accuracy of a displacement measurement of a rough surface could be increased by using a second diffuser as a static pivot point, in that the gearing between speckle and object displacement, cf. Eq. (10), approaches B_2 / B_1. To give a rough estimate of this accuracy we have to know the gearing and the speckle size. Neglecting noise and decorrelation, the accuracy for determining the displacement of a speckle pattern is proportional to the speckle size divided by the square root of the number of speckles. Combining the expressions in Eq. (10) and (12) gives that the accuracy $\delta \mathbf{r}_0$ for determination of $\Delta \mathbf{r}_0$ is proportional to

$$\delta \mathbf{r}_0 \propto \frac{\sqrt{8 B_1^2 + A_1^2 k^2 \rho_1^2 \omega_0^2}}{8 B_1} \rho_1 . \tag{13}$$

Next, we will in Table 1 summarize the results for three generic systems connecting the two diffusers.

Table 1: Speckle properties for generic systems

System	**Matrix element**	Gearing= $\lvert\Delta \mathbf{p}_{peak}\rvert / \lvert\Delta \mathbf{r}_0\rvert$	$\Delta \mathbf{r}_{0,dec}$	$\rho_{speckle}$
Imaging; $G_1(\mathbf{r},\mathbf{q})$	$B_1 \to 0$	0	$\rho_1 / \left(\sqrt{2}\omega_0\right)$	$2 B_2 / \left(A_1 k \omega_0\right)$
Fourier Transf. $G_1(\mathbf{r},\mathbf{q})$	$A_1 \to 0$	B_2 / B_1	ω_0	$\rho_1 B_2 / \left(\sqrt{2} B_1\right)$
Fine grained first diffuser Giving extended illumination of second diffuser, i.e. unphysical	$\rho_1 \to 0$	B_2 / B_1	$\omega_0 \sqrt{\frac{1}{1 + A_1^2 k^2 \omega_0^4 / \left(4 B_1^2\right)}}$	$\to 0$

In the above, we have tacitly assumed the incident field to be a collimated Gaussian beam. In case the field had a finite curvature R_{in}, the matrix elements should be replaced with the new matrix elements A_1', B_1' and D_1':

$$A_1' \to A_1 - B_1 / R_{in},\; B_1' \to B_1 \text{ and } D_1' \to D_1. \tag{14}$$

3. MEASUREMENTS

A series of measurements were conducted with a simple setup. Here, both optical systems, i.e. the one connecting the two diffusers and the one in between the second diffuser and the CMOS array, consisted of free space. This distance (the B_2-element) was held constant at $L_2 = 450\text{mm}$, whereas the distance (the B_1 element) could be varied. In order to obtain negative distances between the two diffusers, the first diffuser was imaged with a large-aperture lens close to the second diffuser, being it in front or behind. The gearing thus, cf. Eq. (10) could attain positive as well as negative values. The first diffuser was illuminated from the backside with a HeNe laser with a wavelength of 633 nm and a spot size $\omega_0 = 0.4$ mm .

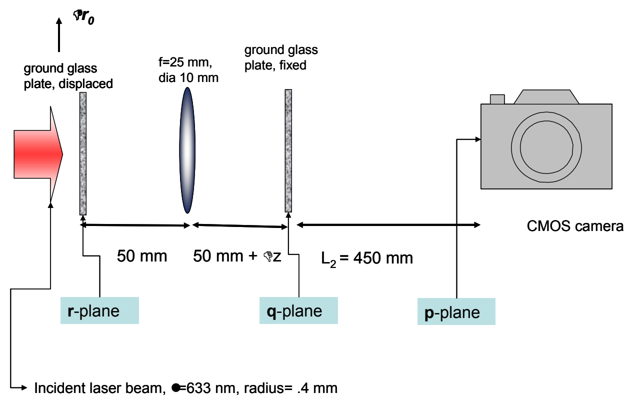


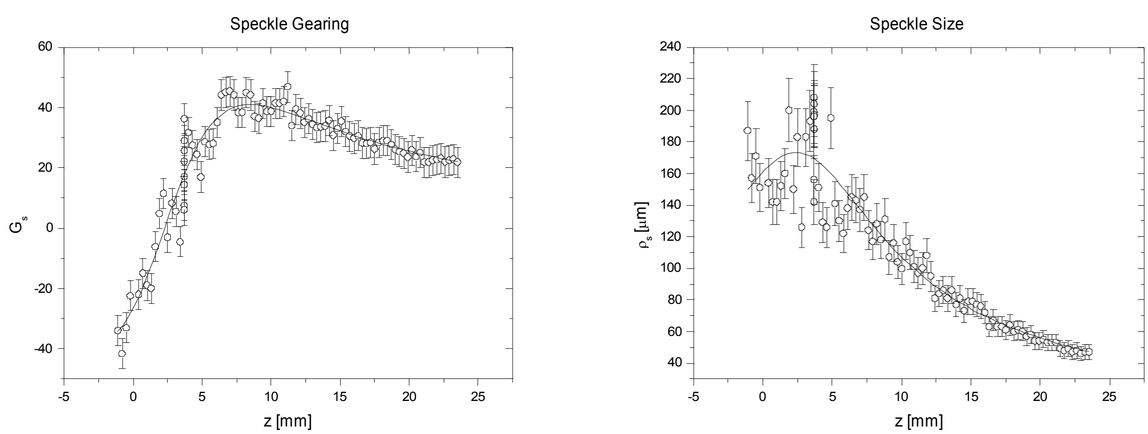
Figure 2. Setup used for simulating varying effective distance between the two diffusers, including negative values.

We had the following governing parameters for the measurements, shown in Table 2:

Table 2. Matrix values for free space propagation experiment

	A	***B* [mm]**	***D***	ρ_1 **[mm]**	ω_0 **[mm]**
$G_1(\mathbf{r},\mathbf{q})$	1	Variable $-3 \le z \le 25$	1	To be determined	.4
$G_2(\mathbf{q},\mathbf{p})$	1	100	1	To be determined	.4

Measurements were conducted by measuring the speckle size, the gearing and the decorrelation as the static diffuser was positioned at either side of the image plane of the dynamic diffuser being displaced in its own plane orthogonally to the optical axis. In this way, measurements could be performed for various effective distances between the two diffusers, even covering negative distances, i.e. negative values of B_1.



a b

Fig.3. The speckle gearing (a) and the speckle size (b) as a function of the distance between the two diffusers. Coinciding diffusers correspond to z = 2.4mm for which value $B_1 = 0$. Theoretical curves are shown in solid lines.

The result of the measurements of speckle gearing (Eq. (10)) and speckle size (Eq. (12)) are shown in Fig. 3. When B_1 equal zero, the image of the first diffuser coincides with the second diffuser, and the gearing becomes zero. Accordingly this corresponds to a *z*-value of 2.3mm in Fig. 3a. At this position, the speckle size 180μm is independent of the effective scale ρ_1 of the first diffuser and is given by $2B_2/(A_1 k\omega_0)$. This gives $\omega_0 = 0.50$mm. Using the value for maximum gearing from Fig. 3a, here 40 can be used to find ρ_1, which becomes 2.9μm.

4. DISCUSSION

General expressions for speckle dynamics including speckle size, speckle gearing and speckle decorrelation have been derived in case a dynamic structure is observed through a fixed scattering surface. The results are given in terms of real-valued *ABCD*-matrix elements, which cover all optical elements having surfaces with transverse quadratic phase variations. The inclusion of Gaussian apodized absorbing apertures would merely call for redo the calculations leading up to Eq. (9), now adhering to the fact that the matrix elements are complex-valued.

The derivation has impact for establishing sensor systems with which the speckle displacement from a moving object can be magnified by using a static diffuser as a “lever arm” when placed close to the dynamic structure. Alternatively, the speckle velocity can be reduced, if so desired. This might facilitate measurement of high speckle velocities, by downscaling. Furthermore, the investigation is of importance when measuring blood flow in tissue, where scattering from the skin surface, *stratum corneum*, will act as the static diffuser. It is customary here to relate speckle dynamics as a reliable measure of blood perfusion. But, as it is shown, both the speckle size and especially the speckle velocity depend highly on the distance between the two structures.

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